Space-Time Network Coding with Optimal Node Selection for Amplify-and-Forward Cooperative Networks

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Abstract—In a wireless network with multiple amplify-and-forward (AF) nodes, the many-to-many cooperative communication is achieved through the novel concept of space-time network coding with optimal node selection (STNC-ONS). The communication under the STNC-ONS scheme is split into two phases: 1) the broadcasting phase (BP) in which each node in its allocated time-slot broadcasts its data symbol to all the other nodes in the network, and 2) the cooperation phase (CP) in which the optimal node corresponding to each desired data symbol is selected for relaying it to the destination node in its own time-slot. The optimal node selection is based on the maximum harmonic mean value of the source, intermediate, and destination nodes' scaled instantaneous channel gains. An approximate symbol-error-rate (SER) expression for M-ary phase shift keying (M-PSK) modulation is derived along with an upper bound SER approximation which is shown to be asymptotic at high signal-to-noise ratio. The derived analytical expressions verify that for a network of \( N \) nodes, a full diversity order of \((N - 1)\) per node is achieved by the STNC-ONS scheme. It is concluded that the STNC-ONS scheme is a potential many-to-many cooperative communication scheme with applications spanning sensor and mobile wireless networks.

Index Terms—Amplify-and-forward (AF), cooperation, harmonic mean, node selection, symbol-error-rate, wireless network coding

I. INTRODUCTION

NETWORK coding has recently emerged as important design paradigm for wireless networks that allows multi-terminal communications and also improves data distribution and network throughput [1]. Cooperative communications has also attracted much attention in the wireless research literature as an effective means of jointly sharing transmissions of distributed single antenna nodes to exploit spatial diversity and mitigate channel fading and interference [2]. As most traditional multinode cooperative communication schemes are not directly applicable to information exchange across many geographically distributed nodes, wireless network coding has become increasingly attractive.

A few recent works have proposed the use of wireless network coding for multinode cooperative communications in wireless networks. For instance, in [3], the concept of wireless network cocast (WNC) that employs wireless network coding is proposed to achieve aggregate transmission power and delay reduction while achieving incremental diversity in location-aware networks. In [4], complex field network coding (CFNC) was employed to achieve a full diversity gain and a throughput as high as 1/2 symbol per user per channel use. However, the communication is limited to \( N_S \) source nodes and a common destination through a single relay or \( N_R \) dedicated relay nodes but not between the source nodes themselves.

Research thus far had not fully exploited the joint potential of wireless network coding and cooperative diversity until the introduction of the novel concept of space-time network coding (STNC) [5][6]. In [5], the multipoint-to-point (M2P) and point-to-multipoint (P2M) space-time network codes were proposed to allow multiple source transmissions to a common node and the reverse common node transmission to multiple destinations, respectively. It was also shown that for a network of \( N \) nodes deploying M2P-STNC or P2M-STNC, only \( 2N \) time-slots are required while achieving a diversity order of \( N \) per transmitted symbol. In [6], the many-to-many space-time network coding (M2M-STNC) for a network of \( N \) decode-and-forward (DF) nodes is proposed to achieve a diversity order of \( N - 1 \) per node over a total of \( 2N \) time-slots while having a stable network throughput of 1/2 symbol per time-slot per node.

In conventional amplify-and-forward (AF) relay networks with \( N \) nodes, \( N - 2 \) nodes act as relays between the source and destination nodes with the available channel resources split into \( N - 1 \) orthogonal channels through TDMA [2]. In this case, \((N - 1)\)-diversity order is achieved (assuming a direct link between the source and destination nodes) [2]. However, the increase in the number of potential relays results in inefficient network bandwidth utilization and gives rise to the necessity of reducing the number of transmissions by selecting the optimal node for relaying signals from source to destination nodes while allowing multiple nodes to communicate simultaneously.

Selection in cooperative networks is not a new concept; however the novelty of this paper is manifested by the use of space-time-network coding to allow \( N \) distributed AF nodes to exchange their data symbols simultaneously while incorporating optimal node selection (i.e. STNC-ONS) and maintaining a stable network throughput of 1/2 symbol per time-slot per node. Approximate analytical symbol error rate (SER) and asymptotic upper-bound performance expressions at high signal-to-noise ratio (SNR) for M-PSK systems along with comparative simulation results are provided. The cooperative diversity order of \( N - 1 \) achievable per node is also verified.

In the remainder of this paper, the system model of the STNC-ONS scheme and the broadcasting and cooperation phases are presented in Section II. The approximate theoretical symbol error rate is analyzed in Section III while the asymptotic upper-bound and diversity order analysis are presented in Section IV. In Section V, the simulation results are contrasted with the analytical results. Finally, conclusions are drawn in Section VI.

II. SYSTEM MODEL

The STNC system model is based on a wireless network with \( N \) single antenna amplify-and-forward nodes denoted as \( S_1, S_2, \ldots, S_N \) for \( N \geq 4 \). Each node \( S_j \) for \( j \in \{1, 2, \ldots, N\} \) is assumed to have its own data symbol \( x_j \) to
exchange with all the other \(N - 1\) nodes in the network. In this paper, the channel between any two nodes is modeled as flat Rayleigh fading with additive white Gaussian noise (AWGN). Let \(h_{j,i}\) denote a generic channel coefficient representing the channel between any two nodes \(S_j\) and \(S_i\) for \(j \neq i\) and \(h_{j,i}\) is modeled as a zero-mean complex Gaussian random variable with variance \(\sigma^2_{j,i}\) (i.e., \(h_{j,i} \sim \mathcal{CN}(0, \sigma^2_{j,i})\)), where the channel variance \(\sigma^2_{j,i} = d_{j,i}^{-\nu}\) with \(d_{j,i}\) and \(\nu\) being the distance between the two nodes and the path-loss exponent, respectively. The channel gain \(|h_{j,i}|^2\) is modeled as a Rayleigh random variable while the channel gain squared \(|h_{j,i}|^2\) is modeled as an exponential random variable with mean \(\sigma^2_{j,i}\). Also, the channel gains \(h_{j,i}\) between nodes \(S_j\) and \(S_i\) is assumed to be reciprocal (i.e., \(h_{i,j} = h_{j,i}\)) as in time division duplexing (TDD) systems, with perfect channel estimation at each node. Moreover, the channel coefficients are assumed to be quasi-static throughout the network operation. Finally, perfect synchronization between all the \(N\) nodes in the network is assumed.

The cooperative communication between all the nodes (depicted in Fig. 1 for \(N = 4\)) is performed over a total of \(2N\) time-slots and is split into two phases (\(2N\) time-slots each): a) the broadcasting phase (BP) and b) the cooperation phase (CP), as discussed in the following subsections.

A. Broadcasting Phase

In the broadcasting phase, source node \(S_j\) is assigned a time-slot \(T_j\) in which it broadcasts its own data symbol \(x_j\) to the \(N - 1\) other nodes \(S_i\) in the network for \(i \in \{1, 2, \ldots, N\}\) for \(i \neq j\), which can be expressed in matrix form as follows [6]

\[
\begin{bmatrix}
S_1 & \cdots & S_j & \cdots & S_N \\
T_1 & x_1 & \cdots & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
T_j & 0 & \cdots & x_j & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
T_N & 0 & \cdots & 0 & \cdots & x_N
\end{bmatrix}
\]  

(1)

For source separation for each of the transmitted symbols of the different nodes at each receiving node, each symbol \(x_j\) is spread using a signature waveform \(s_j(t)\) where it is assumed that each node knows the signature waveforms of all the other nodes. The cross-correlation of \(s_j(t)\) and \(s_i(t)\) is \(\rho_{j,i} = \langle s_j(t), s_i(t) \rangle = (1/T) \int_0^T s_j(t)s_i^*(t)dt\) for \(j \neq i\) with \(\rho_{i,j} = 1\) and \(T\) being the symbol duration. Thus, the signal received at node \(S_i\) for \(i \neq j\) in time-slot \(T_j\) is expressed as

\[
y_{j,i}(t) = \sqrt{P_j^B} h_{j,i} x_j s_j(t) + n_{j,i}(t),
\]

(2)

where \(P_j^B\) is the transmit power in the broadcasting phase at node \(S_j\) and \(h_{j,i}\) is the Rayleigh flat fading channel coefficient between nodes \(S_j\) and \(S_i\). Also, \(n_{j,i}(t)\) is the AWGN sample at node \(S_i\) due to the signal transmitted by node \(S_j\), modeled as a zero-mean complex Gaussian random variable with variance \(N_0\). To extract data symbol \(x_j\) at node \(S_i\), the received signal \(y_{j,i}(t)\) is cross-correlated with the signature waveform \(s_j(t)\) to obtain

\[
y_{j,i}(t) = \langle y_{j,i}(t), s_j(t) \rangle = \sqrt{P_j^B} h_{j,i} x_j + n_{j,i},
\]

(3)

where \(n_{j,i} \sim \mathcal{CN}(0, N_0)\). Upon completion of the broadcasting phase, each node \(S_i\) will have exchanged its data symbol \(x_i\) with the other nodes and received a set of \((N - 1)\) signals \(\{y_{j,i}\}_{j \neq i}\) comprising symbols \(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_N\) for \(j \neq i\) from all the other nodes in the network. Node \(S_i\) then performs a matched filtering operation on each of the received signals \(y_{j,i}\) and the SNR at the output of the matched-filter is expressed as [2]

\[
r_{j,i}^{BP} = \frac{P_j^B |h_{j,i}|^2}{N_0}.
\]

(4)

The received signals at all the nodes in the network at the end of the broadcasting phase can be expressed in a matrix form as

\[
Y = \begin{bmatrix}
y_{1,2} & \cdots & y_{N-1,1} & y_{N,1} \\
y_{1,2} & \cdots & y_{N-1,2} & y_{N,2} \\
\vdots & \ddots & \vdots & \vdots \\
y_{1,N-1} & y_{2,N-1} & \cdots & y_{N-1,N} \\
y_{1,N} & y_{2,N} & \cdots & y_{N-1,N}
\end{bmatrix},
\]

(5)

where the \(i^{th}\) row represents the signals received at \(S_i\) while the \(j^{th}\) column represents the signals received in time-slot \(T_j\) from \(S_j\).

B. Cooperation Phase

In the cooperation phase, each node \(S_i\) acts as the destination node in time-slot \(T_{N+i}\) for \(i \in \{1, 2, \ldots, N\}\) and receives simultaneous transmissions from the other \(N - 1\) nodes. Each time-slot \(T_{N+i}\) involves two operations: 1) signal transmission via optimally selected nodes, and 2) multinode signal detection, which are discussed in the following subsections, respectively.

1) Signal Transmission via Optimally Selected Nodes: When node \(S_i\) acts as a destination node in its assigned time-slot \(T_{N+i}\), the intermediate node the transmitted signal of which results in the highest cumulative SNR value for symbol \(x_m\) for \(m \neq i\) is utilized. The node selection metric used by the destination node \(S_i\) to determine the optimal node \(S_k\) to “relay” symbol \(x_m\) received from source node \(S_m\) for \(k \neq i\) and \(k \neq m\) is based on the scaled harmonic mean of the instantaneous source, intermediate and destination nodes’ scaled channel gains, as follows [8] [9] [10]

\[
\tilde{\gamma}_{m,k,i} \triangleq \bar{\mu}_H(x_m^B, x_{m,k}^C) = x_m^B x_{m,k}^C/(x_m^B + x_{m,k}^C),
\]

(6)

where \(\bar{\mu}_H\) denotes a control messages exchange prior to the cooperation phase [7] and is only updated when the respective channels’ coherence time elapses.
where $X_{m,k}^B = P_m^{B}|h_{m,k}|^2$ and $X_{m,k}^C = P_m^{C}|h_{m,k}|^2$ are exponential random variables corresponding to the broadcast transmission of symbol $x_m$ from source node $S_m$ to intermediate node $S_k$ with transmit power $P_m^B$ and the cooperative transmission of symbol $x_m$ from intermediate node $S_k$ to the destination node $S_i$ with transmit power $P_m^C$. Thus, the scaled harmonic mean values corresponding to symbol $x_m$, for $m \neq i$ at node $S_k$ for $k \neq i$ and $k \neq m$ when node $S_i$ is the destination node is summarized in matrix form as

$$
\mathbf{r}_i = \begin{bmatrix}
\gamma_{1,i,1,i} & \gamma_{1,i,1,i} & \cdots & \gamma_{1,i,1,i} \\
\vdots & \ddots & \ddots & \vdots \\
\gamma_{1,i,1,N,i} & \gamma_{1,i,1,N,i} & \cdots & \gamma_{1,i,1,N,i}
\end{bmatrix}
$$

For node $S_i$ to receive symbol $x_m$ for $m \neq i$, the optimally selected node $S_{m,opt,i}$ to forward symbol $x_m$ among the $N - 2$ nodes that received independent copies of symbol $x_m$ during the broadcasting phase is defined by

$$
k_{m,i} = \arg \max_{k=1,2,\ldots,N} \{ \gamma_{m,k,i} \} \quad k \neq m.
$$

Hence, in time-slot $T_{N+i}$ for each symbol $x_m$ for $m \neq i$, the system reduces to a source node $S_m$, a destination node $S_i$ and an optimal node $S_{m,opt,i}$ for the transmission of $x_m$. Thus, each symbol $x_m$ is associated with a set of indicator functions in the form of $\mathcal{I}_{m,i} = \{ h_{m,k}^{opt,i} \}^N_{k=1,k \neq m}$, where $h_{m,k}^{opt,i}$ for $k \neq i, k \neq m$ acts as a binary indicator function when node $S_i$ is the receiving node; while $S_k$ is the optimal node $S_{m,opt,i}$ transmitting signal $y_m,k$ corresponding to symbol $x_m$. Hence, $\mathcal{I}_{m,k,i}$ is defined by $\mathcal{I}_{m,k,i} = 1$ if $k = m^{opt,i}$, otherwise $\mathcal{I}_{m,k,i} = 0$. Each node $S_k$ then possibly forms a linearly-coded signal $X_k^c(t)$ from its received signals in the broadcasting phase and transmits it to node $S_i$ during time-slot $T_{N+i}$. Specifically, $X_k^c(t)$ is composed from the received signals of the $kth$ row of matrix $\mathbf{Y}$ in (5) in the form of

$$
X_k^c(t) = \sum_{m=1}^{N} \beta_{m,k,i} y_{m,k} \mathcal{I}_{m,k,i} s_m(t) \tag{8}
$$

where $s_m(t)$ is the signature waveform associated with the symbol $x_m$ and $\beta_{m,k,i}$ is a scaling factor defined as [2]

$$
\beta_{m,k,i} = \sqrt{P_m^{C}/(P_m^{B}|h_{m,k}|^2 + N_0)}. \tag{9}
$$

It should be noted that if node $S_k$ is not an optimal node to forward any of the $x_m$ for $m \neq i, m \neq k$ data signals to node $S_i$, then $X_k^c(t) = 0$; otherwise, node $S_k$ is an optimal node to forward at least one symbol $x_m$ and $X_k^c(t) \neq 0$. Hence, the signal transmission of the cooperation phase can be expressed in matrix form as [6]

$$
\begin{array}{c}
S_1 \\
S_i \\
S_N
\end{array}
= \begin{bmatrix}
T_{N+i} & 0 & \cdots & \cdots & X_1^c & \cdots & X_N^c \\
\cdots & \cdots & \ddots & \cdots & \vdots & \cdots & \vdots \\
\cdots & \cdots & \cdots & \ddots & \vdots & \cdots & \vdots \\
T_{N+i} & X_1^c & \cdots & \cdots & X_1^c & \cdots & X_N^c \\
\cdots & \cdots & \vdots & \ddots & \vdots & \cdots & \vdots \\
T_{2N} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0
\end{bmatrix}
\tag{10}
$$

The received signal at node $S_i$ during time-slot $T_{N+i}$ is given by

$$
Y_i(t) = \sum_{k=1 \text{ and } k \neq i}^{N} h_{k,i} X_k^c(t) + w_i(t) \tag{11}
$$

$$
= \sum_{m=1 \text{ and } m \neq i}^{N} \alpha_{m,i} \sqrt{P_m^B |h_{m,opt,i}| x_m s_m(t) + \bar{w}_i(t)},
$$

where $h_{m,opt,i}$ is the channel coefficient between the optimally selected node and node $S_i$ (i.e. $h_{opt,i}$) for the transmission of symbol $x_m$ for $m \neq i$. Furthermore, $\alpha_{m,i}$ is defined as

$$
\alpha_{m,i} = \sum_{k=1 \text{ and } k \neq i, k \neq m}^{N} \beta_{m,k,i} \mathcal{I}_{m,k,i} h_{m,k} = \beta_{m,opt,i} h_{m,opt,i}, \tag{12}
$$

with $h_{m,opt,i}$ being the channel coefficient between the source node $S_m$ and the optimally selected node (i.e. $h_{opt}$) to forward symbol $x_m$ to node $S_i$ for $m \neq i$, as implied by $k = m^{opt,i}$, and $\beta_{m,opt,i}$ is the scaling factor defined in (9). Moreover, in (11), $w_i(t)$ is the zero-mean $N_0$-variance AWGN sample at node $S_i$ and $\bar{w}_i(t)$ is the equivalent noise term which can be expressed as

$$
\bar{w}_i(t) = w_i(t) + \sum_{m=1 \text{ and } m \neq i}^{N} \beta_{m,opt,i} h_{m,opt,i} n_{m,opt,i} s_m(t), \tag{13}
$$

where $n_{m,opt,i}$ is the noise sample at the optimally selected node by node $S_i$ for the transmission of symbol $x_m$, for $m \neq i$.

The total power $P_{m}^{i}$ associated with transmitting symbol $x_m$ from node $S_m$ to node $S_i$ is distributed between the broadcasting and cooperation phases (i.e. $P_{m}^{i} = P_m^B + P_m^C$, with $P_m^B = 6 \delta m_i$ and $P_m^{opt,i} = (1 - \delta) P_{m_i}$ and $\delta$ being the power ratio factor such that $0 < \delta \leq 1$). For notational convenience and without any loss of generality, it is assumed that all the transmitted symbols between any nodes $S_m$ and $S_i$ have the same total power allocation (i.e. $P_{m}^{i} = P = P_m^B + P_m^{opt,i}$, $\forall m, i \in \{ 1, 2, \ldots, N \}$ and $m \neq i$).

2) Multinode Signal Detection: Upon receiving signal $Y_i(t)$, a multinode signal detection operation is performed by node $S_i$ to extract each of the $(N - 1)$ symbols $x_j$, for $j \in \{ 1, 2, \ldots, N \} \neq i$. This is achieved by passing the received signal $Y_i(t)$ through a matched filter bank (MFB) of $(N - 1)$ branches, matched to the corresponding nodes’ signature waveforms $y_{i,j}(t)$, yielding

$$
Y_{i,j} = Y_i(t, s_j(t)) = \sum_{m=1 \text{ and } m \neq i}^{N} \alpha_{m,i} \sqrt{P_m^B |h_{m,opt,i}| x_m p_{m,j} + \bar{w}_j}, \tag{14}
$$

where $p_{m,j}$ is the correlation coefficient between $s_m(t)$ and $s_j(t)$. The output of the MFB can be put in a vector form of all the $(N - 1)$ $Y_{i,j}$’s signals as follows

$$
\mathbf{y}_i = [Y_{i,1}, Y_{i,2}, \ldots, Y_{i,N}]^T, \tag{15}
$$

$$
\mathbf{x}_i = [x_1, x_2, \ldots, x_N]^T, \tag{16}
$$

$$
\bar{w}_i = [\bar{w}_1, \bar{w}_2, \ldots, \bar{w}_N]^T \sim \mathcal{CN}(0, N_0 B_x R_i), \tag{17}
$$

where $R_i$ and $R_i$ are $(N - 1) \times (N - 1)$ with $R_i$ being defined as
while the diagonal matrices \( A_i \) and \( B_i \) are respectively written as

\[
A_i = \text{diag} \left[ \bar{o}_i, 1, \bar{o}_i, \bar{o}_i, \ldots, \bar{o}_i, \bar{o}_i \right],
\]

and

\[
B_i = \text{diag} \left\{ 1 + \beta_{i,1}^2, 1 + \beta_{i,2}^2, \ldots, 1 + \beta_{i,N_i}^2 \right\},
\]

with \( \bar{o}_{j,i} \) and \( \beta_{j,i}^2 \) being defined as \( \bar{o}_{j,i} = \alpha_{j,i} \sqrt{P_j^B h_{j,i}} \) and \( \beta_{j,i} = \left( \bar{o}_{j,i} h_{j,i} \right)^2 \) for \( j \neq i \), respectively. The received signal vector \( \tilde{Y}_{j,i} \) can then be decorrelated (assuming matrix \( R_i \) is invertible) as \( \tilde{Y}_{j,i} = R_i^{-1} Y_i = A_i x_i + \tilde{w}_i \), where \( \tilde{w}_i \) is \( R_i^{-1} \tilde{w}_i \) and \( \tilde{w}_i \sim \mathcal{CN}(0, N_0 B_k R_i^{-1}) \). Thus, at node \( S_i \), the decorrelated received signal \( \tilde{Y}_{j,i} \) corresponding to symbol \( x_j \) is obtained as

\[
\tilde{Y}_{j,i} = \beta_{j,i} h_{j,i} x_j + \tilde{w}_{j,i},
\]

where \( \tilde{w}_{j,i} \sim \mathcal{CN}(0, N_0 (1 + (\beta_{j,i} h_{j,i})^2)^{r_{j,i}}) \) and \( r_{j,i} \) is the \( j^\text{th} \) diagonal element of matrix \( R_i^{-1} \). Without loss of generality, it is assumed that \( \rho_{j,i} = \rho \) for all \( j \neq i \) and thus

\[
r_{j,i} = \frac{1 + (N - 3)\rho}{1 + (N - 2)\rho} \Delta_{N-1}.
\]

It should be noted that upon the completion of the broadcasting and cooperation phases, each node \( S_i \) for \( i = 1, 2, \ldots, N \) has received two signals containing symbol \( x_j \) for \( j = 1, 2, \ldots, N \) and \( j \neq i \); a direct signal from the source node \( S_j \) in the broadcasting phase and an optimally selected signal among the \((N-2)\) nodes \( S_m \) for \( m \neq i \) and \( m \neq j \), in the cooperation phase. Thus, over the two phases, each node acts as a source, a relay and a destination, alternatively.

The detection of symbol \( x_j \) at the node \( S_i \), denoted as \( \hat{x}_{j,i} \), can be achieved through maximal-ratio-combining (MRC) of the signals received in the broadcasting and cooperation phases as [2]

\[
\hat{x}_{j,i} = \frac{\sqrt{P_j^B} y_{j,i} + \beta_{j,i} h_{j,i} x_j}{\sqrt{1 + \left( \beta_{j,i} h_{j,i} \right)^2}} \tilde{Y}_{j,i},
\]

where \((\cdot)^*\) denotes complex conjugation while \( y_{j,i} \) and \( \tilde{Y}_{j,i} \) follow (3) and (21), respectively. Thus, the instantaneous SNR at the output of the MRC at node \( S_i \) corresponding to symbol \( x_j \) is given by

\[
\gamma_{j,i} = \gamma_{BP,i} + \gamma_{CP,i},
\]

where \( \gamma_{BP,i} \) is an exponential random variable as in (4) with mean \( \lambda_{BP} = \frac{N_0}{\sqrt{P_j^B}} \). Also, it is easily verified that \( \gamma_{CP,i} \) is given by

\[
\gamma_{CP,i} = \frac{P_{opt,C}^C P_{j,i}^B (h_{j,i})^2}{N_0 \rho_{N-1}^2 (P_{j,i}^B (h_{j,i})^2 + P_{opt,C}^C (h_{j,i})^2 + 1)},
\]

which at high SNR can be tightly approximated as [2]

\[
\gamma_{j,i} \approx \frac{P_{opt,C}^C P_{j,i}^B (h_{j,i})^2}{N_0 \rho_{N-1}^2 (P_{j,i}^B (h_{j,i})^2 + P_{opt,C}^C (h_{j,i})^2 + 1)}.
\]
achieves a full diversity order of $P/N$.

Hence, by substituting (37) and (39) into (30), the upper-bound (by ignoring the 1 term) as 

$$P_{\text{SER}}(\gamma^*) \leq \left( \frac{N_0}{P} \right)^{N-1} \left( \frac{N-3}{\sigma_s^2} \right)^{N-2} \sum_{m=1}^{N} \sum_{k=1, k \neq m, k \neq j}^{N} \Phi(j, m, i) \Phi(j, k, i).$$

(40)

Therefore, the PDF of $\gamma^*_{j,i}$ can be obtained as

$$p_{\gamma^*_{j, i}}(\gamma) = \sum_{m=1, m \neq i, m \neq j}^{N} p_{\gamma^*_{j, m, i}}(\gamma) \prod_{k=1, k \neq m, k \neq j}^{N} \left( 1 - e^{-\gamma \lambda_{BC}^{j,k,i}} \right),$$

(34)

where $p_{\gamma^*_{j, m, i}}(\gamma) = \lambda_{BC}^{j,m,i} e^{-\gamma \lambda_{BC}^{j,m,i}}$ is the PDF of $\gamma^*_{j, m, i}$. Using (34) to determine the MGF of $\gamma^*_{j,i}$ is quite difficult [9]; however, by using the relationship between the CDF of a random variable $X$ and its MGF given by $M_X(s) = s \mathcal{L}(P_X(x))$ with $\mathcal{L}$ being the Laplace Transform of the parameter [12], the MGF of $\gamma^*_{j,i}$ can be shown to be as in (35) (top of page). Thus, by substituting (31) and (35) into (30), the approximate SER performance for symbol $x_j$ detected at node $S_i$ (for $i \neq j$) can be determined using (36) (top of page).

IV. UPPER BOUND SER AND DIVERSITY ORDER ANALYSIS

An upper bound on the SER performance is derived by first noticing that at high SNR, the MGF of $\gamma^*_{j,i}$ given in (31) can be upper-bounded (by ignoring the 1 term) as [2]

$$M_{\gamma^*_{j,i}} \leq \frac{N_0}{\sigma_s^2} P_{\gamma^*_{j,i}}.$$ (37)

Second, an upper-bound for $M_{\gamma^*_{j,i}}(s)$ at high SNR can be determined by using the approximation $e^x \approx (1 + x)$ as $x \to 0$ in the PDF of $\gamma^*_{j,i}$ defined in (34) which can then be written as

$$p_{\gamma^*_{j,i}}(\gamma) \approx \sum_{m=1, m \neq i, m \neq j}^{N} \lambda_{BC}^{j,m,i} \left( 1 - \gamma \lambda_{BC}^{j,m,i} \right)^{\gamma-3} \prod_{k=1, k \neq m, k \neq j}^{N} \lambda_{BC}^{j,k,i}.$$ (38)

Since $\lambda_{BC}^{j,k,i} = \frac{N \sigma_r^2 - 1}{P} \Phi(j, k, i)$, so by substituting (38) into (29) and a series of manipulations, it can be shown that

$$M_{\gamma^*_{j,i}}(s) \leq (N-3)! \left( \frac{N \sigma_r^2 N-1}{s P} \right)^{N-2} \sum_{m=1, m \neq i, m \neq j}^{N} \Phi(j, m, i) \prod_{k=1, k \neq m, k \neq j}^{N} \Phi(j, k, i).$$ (39)

Hence, by substituting (37) and (39) into (30), the upper-bound SER expression is obtained as shown in (40) (top of next page) with $\Theta(N-1)$ being defined as $\Theta(N-1) = \frac{1}{P} \int_{0}^{\pi} (M-1)!/M(\sin^2(\theta))^{N-1} d\theta$. The diversity order is defined as $\Gamma = -\lim_{SNR \to \infty} \log(P_{\text{SER}})/\log(SNR)$, where $SNR = P/N_0$ [2]. Thus, it is easily verified that the STNC-ONS achieves a full diversity order of $\Gamma = N - 1$ per node.

The superiority of the STNC-ONS scheme with $N$ nodes manifests itself in the fact that two time-slots are required per node to achieve a diversity order of $N - 1$ per transmitted symbol while allowing all the $N$ nodes to exchange their data symbols simultaneously over a total of $2N$ time-slots as opposed to the conventional multinode relay networks that require $N^2$ time-slots to achieve the same diversity order of $N - 1$ per node even with optimal relay selection [13].

V. SER PERFORMANCE EVALUATION

In this section, the QPSK SER performance of the STNC-ONS for symbol $x_j$ received at node $S_i$ is simulated and compared with the derived approximate and upper-bound SER expressions for $N = 4$ and $N = 5$. For simplicity, equal power allocation between the two transmission phases is assumed (i.e. $\delta = 1/2$ and thus $P_B = P^C_{opt, A} = P/2$). The network under consideration assumes nodes are located as shown in Fig. 2 with the channel variance between nodes $S_j$ and $S_i$ being defined as $h_{j,i} \sim CN(0, d_{j,i}^{-\nu})$, $\forall j, i \in \{1, 2, \ldots, N\}$ and $\nu = 3$. Non-orthogonal signature waveforms with a cross-correlation of $p_{j,i} = \rho = 0.5$ for $j \neq i$ are also assumed.

It is clear from Fig. 3 that as the number of cooperating nodes in the network $N$ increases, the SER performance improves which is due to the increased diversity order achievable with the increase in $N$. Moreover, it is evident that the derived approximate SER theoretical expression coincide with the simulated performance except for a slight deviation at low SNR which is attributed to the approximation used in the theoretical analysis which assumed high enough SNR. Also, the derived upper-bound happens to be asymptotic at high SNR and thus confirms the achievable diversity order per node.

VI. CONCLUSIONS

In this paper, the STNC-ONS scheme is presented and analyzed. In particular, it was shown that the STNC-ONS
scheme allows $N$ amplify-and-forward nodes to exchange their data symbols simultaneously over a total of $2N$ time-slots while achieving a diversity order of $N - 1$ per node. This in turn establishes that the STNC-ONS scheme is much more bandwidth efficient than conventional point-to-point multinode relay networks. Moreover, analytical approximate and asymptotic upper-bound SER expressions for an arbitrary number $N$ of nodes were derived and shown to coincide with the simulated results. Due to the merits of the STNC-ONS scheme, its potential applications include but not limited to cluster-based communications for cooperative spectrum sensing and decision fusion in cognitive radio networks [14], and also for reliable and energy-efficient inter- and intra-cluster data gathering in wireless sensor networks [15]. Moreover, the STNC-ONS scheme can be used for improved network connectivity in clustered mobile ad-hoc networks [16]. Hence, it is concluded that the STNC-ONS scheme serves as a potential many-to-many cooperative communication scheme for amplify-and-forward based wireless networks.

REFERENCES
